Lesson 11

Thermomechanical Measurements for Energy Systems (MENR)

Measurements for Mechanical Systems and Production (MMER)

A.Y. 2015-16

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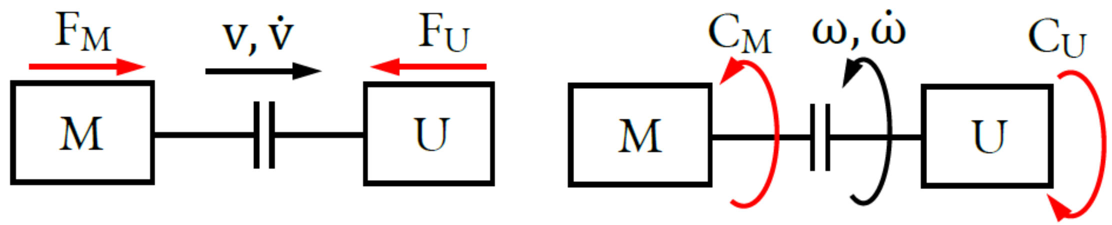
VIBRATION measurement techniques …

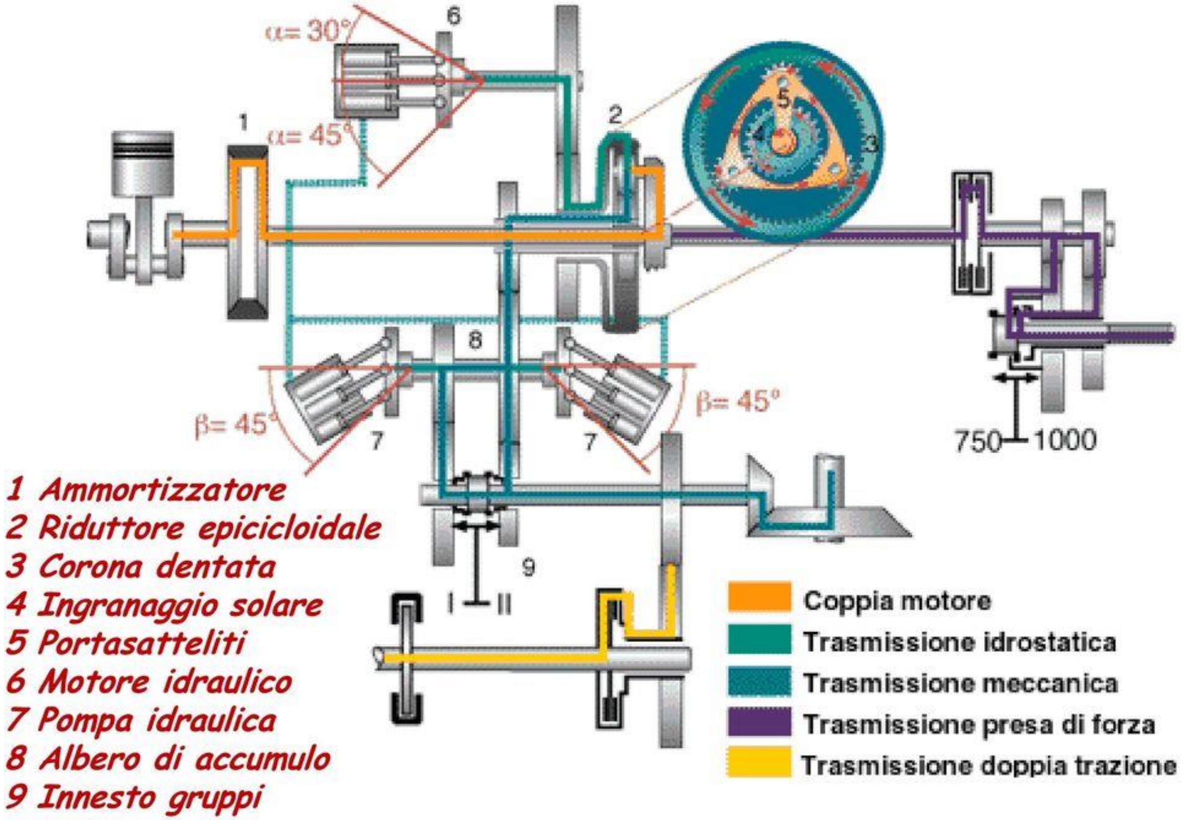


Engines (M) that produce mechanical energy are always coupled with an **Operating Machine** (U) through a shaft, that transfers the energy by an alternating (v) or a rotating (ω) movement.

Relative movements of mechanical parts of engines and operating machines always produce spurious ***vibration*** and audible ***noise***.

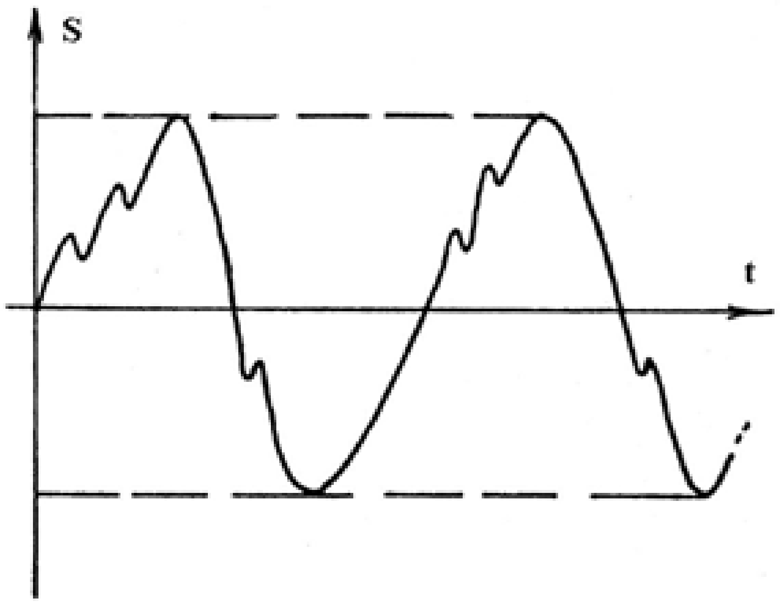
Vibrations are a "mechanical energy waste", are troublesome for operators and harmful to machines, because of the "mechanical fatigue effects" they cause !

example of an agricultural tractor engine:



VIBRATION measurement techniques …

Vibrations are *harmonic motions of mechanical parts* of a machine which, sometimes, can be seen by eye and always, can be heard as noise …



Therefore, **vibrations** are characterized by ***waveforms*** which have their specific ***amplitude*** and ***frequency*** !

Every ***periodic waveform*** can be described by a ***Fourier series*** :

$$x(t) = x(t + 2\pi) = \frac{1}{2} a_0 + \sum_{k=1}^{\infty} \left(a_k \cos kt + b_k senkt \right)$$

therefore, to study vibrations we can refer to its ***component waves***

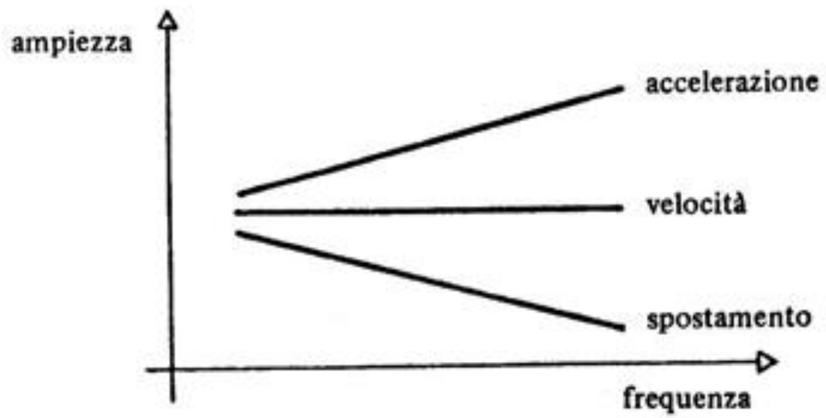
Displacement: $x(t) = Xsen\omega t = Xe^{j\omega t}$

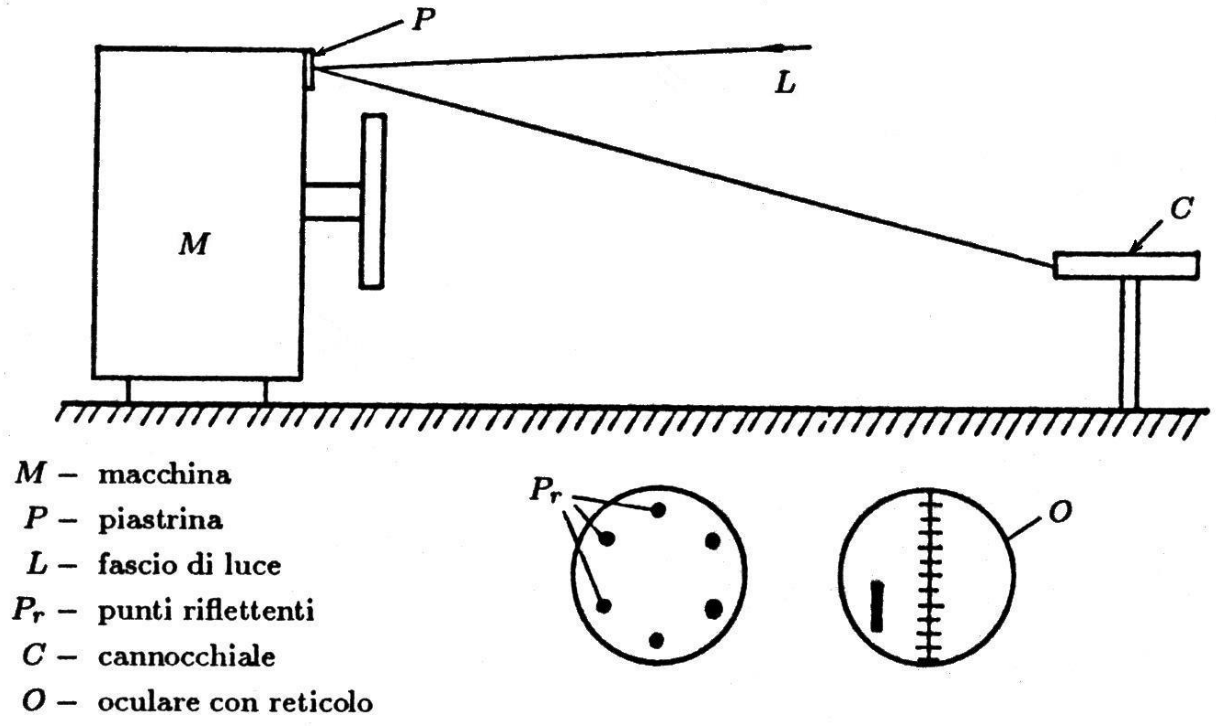
Velocity: $v(t) = X\omega \cos \omega t = X\omega e^{j\omega t}$

Acceleration: $a(t) = -X\omega^2 sen\omega t = -X\omega^2 e^{j\omega t}$

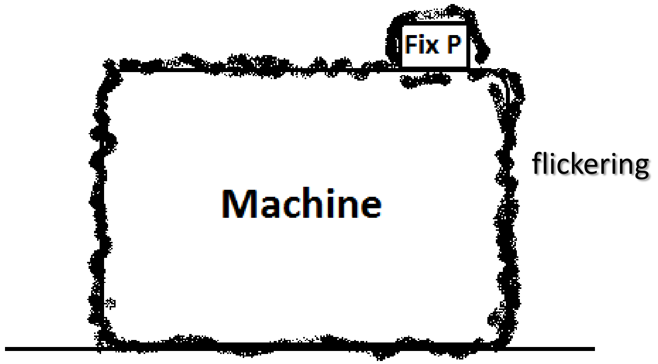
As for a mechanical system in vibration, the momentum $Q = m \cdot v = \cos t$ remains constant, these three simple relations indicate that it is preferable to ***measure displacements*** at *low frequencies* and to ***measure acceleration*** at *high-frequency* !!

VIBRATION measurement techniques …





The main problem for *vibration measurements* is finding a ***fixed point*** that can be considered as *reference point* for all the points or elements of the "vibrating machine" !

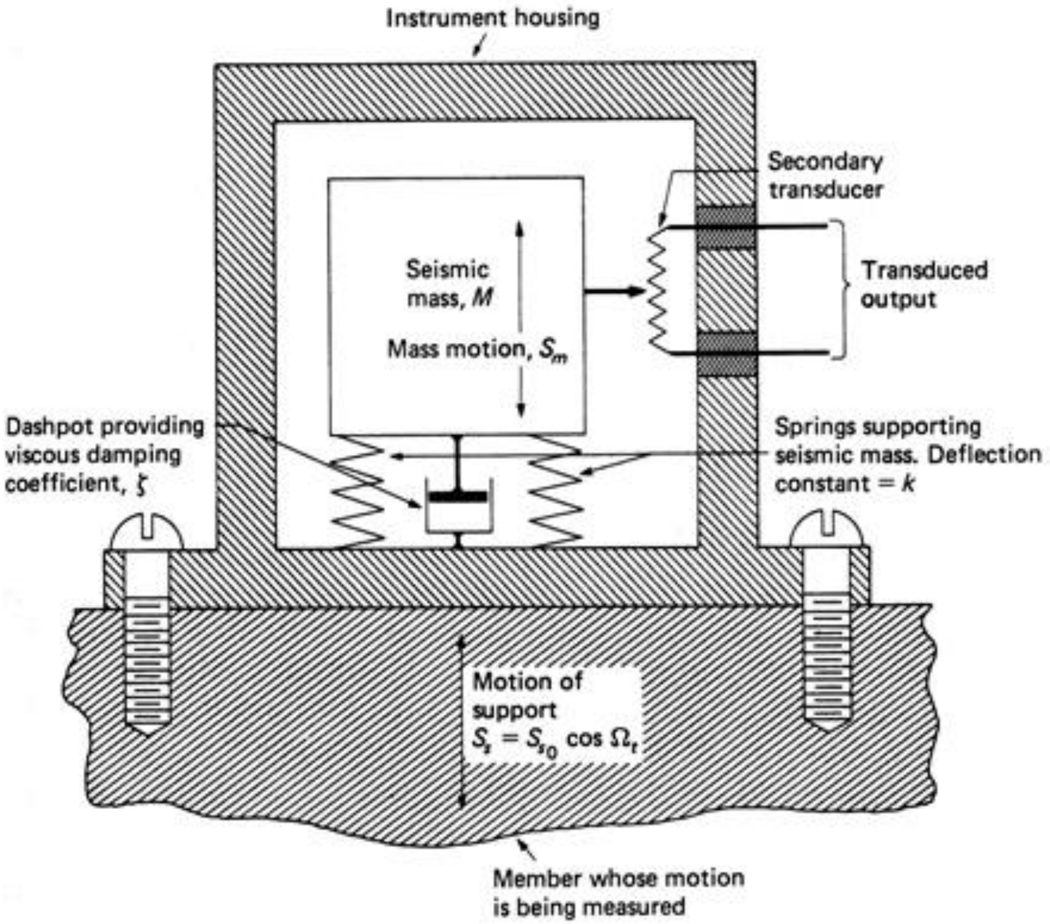


Sometimes, this can be done placing a *telescope instrument* equipped with a *graduated aim* quite far form the vibrating machine !

However, most of the times in industrial plants, machines are placed indoor in a shed !

The ***fixed point**,* therefore, can *NOT BE FAR* from the vibrating machine !

VIBRATION measurement techniques …

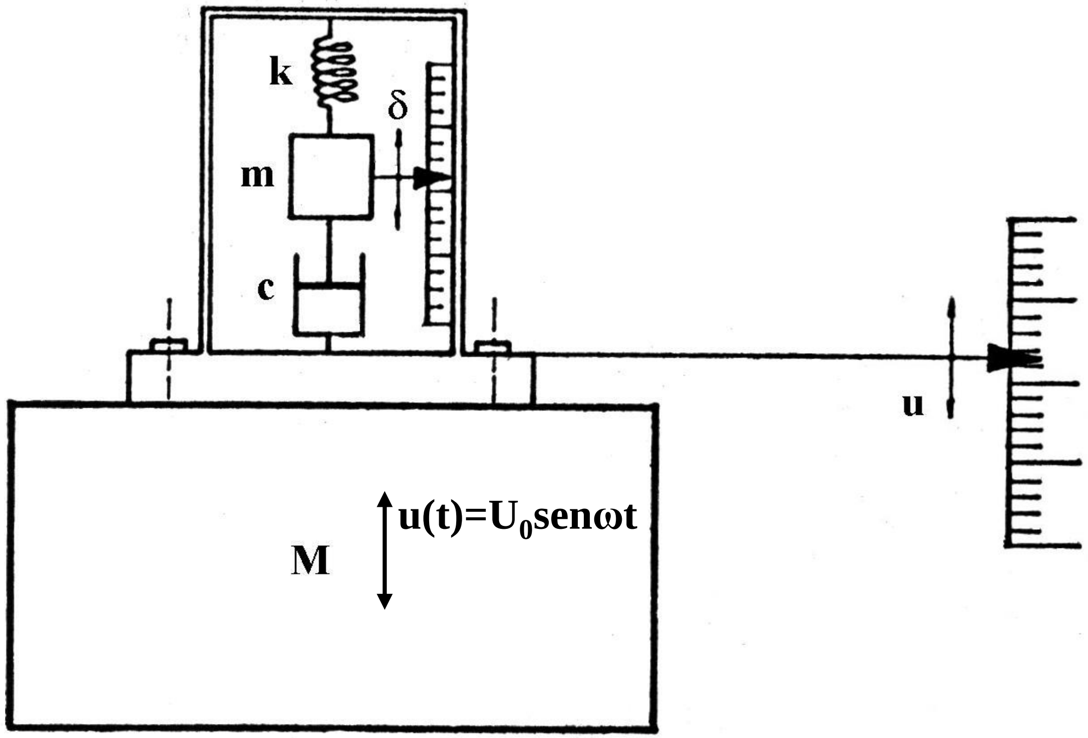


A ***seismic mass*** *m* is placed inside the transducer, connected to the case by means of a *spring k* and a *viscous damper c* …

Seismic transducers utilize an "internal fixed reference point" and are *rigidly attached* to the engine !

How are these sensors actually working … ?

VIBRATION measurement techniques …



We would like to measure the **periodic displacement** ***u(t) = U_0 senωt*** of the machine M, indicated in the figure on the imaginary absolute (fixed) scale !

However, the transcducer output is the **displacement δ(t)** of the *inner seismic mass* **«*m*»** with respect to the sensor external case, indicated in the figure on the real relative (moving) scale …

We can express the *motion of the seismic mass m* by means of the *theorem of relative motion*

$$x(t) = u(t) + \delta(t)$$

- ***x(t)*** ***absolute displacement*** (measured in an inertial reference frame)
- ***u(t)*** ***dragging displacement*** (of the relative reference frame)
- ***δ(t)*** ***relative displacement*** (measured in the relative reference frame)

VIBRATION measurement techniques …

Vibrations are definitely a *dynamic phenomenon* so, we will study first the ***dynamic response of the seismic transducer***: that means finding the ***gain*** δ_0/U_0 with respect to the ***frequency*** ω of the ***displacement u(t)*** …

Seismic transducers are clearly *second order instruments*, so the equation for the dynamic response is :

$$m\frac{d^2x}{dt^2}+c\frac{d\delta}{dt}+k\delta=0$$

because *inertial forces* are always referred to the *inertial reference frame* (x), while *elastic and damping forces* here are referred to the *relative moving frame* (δ) made of the sensor case !

$$m\frac{d^2}{dt^2}(\delta+u)+c\frac{d\delta}{dt}+k\delta=0$$

$$m\frac{d^2\delta}{dt^2}+c\frac{d\delta}{dt}+k\delta=-m\frac{d^2u}{dt^2}=-m(-\omega^2U_o sen\omega t)=m\omega^2u(t)$$

the "*external force term*" in the equation of this transducer is proportional to the *relative displacement u(t)* … that is *$F(t) = m\omega^2U_0\, sen\omega t$*

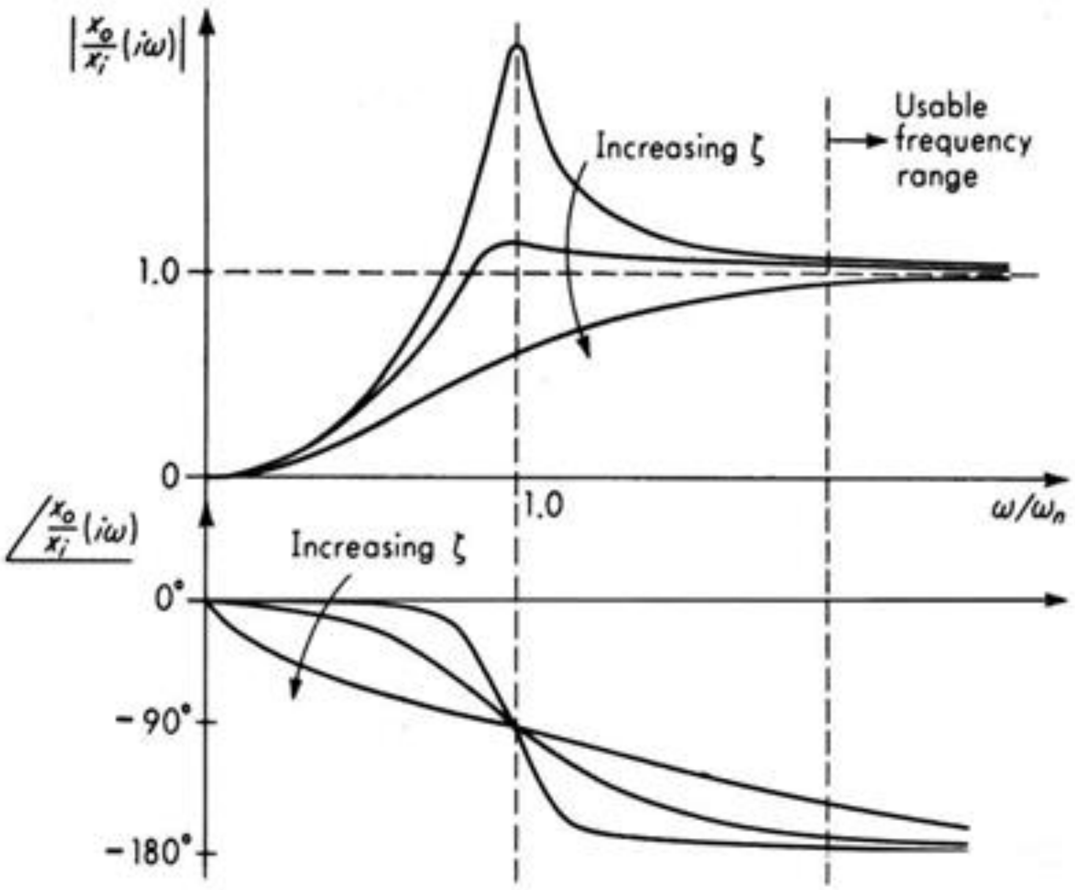
Frequency response is much similar to any other *second order instrument* :

$$X_0=\frac{F_o}{k}\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

VIBRATION measurement techniques …

where : $\frac{F_o}{k} = \frac{m\omega^2 U_o}{k} = \frac{\omega^2 U_o}{k/m} = \frac{\omega^2 U_o}{\omega_n^2}$ and for the amplitude δ_0 :

$$\delta_o = U_o \frac{\omega^2}{\omega_n^2} \cdot \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2 \left(\frac{\omega}{\omega_n}\right)^2}}$$



Seismic-displacement-pickup frequency response.

which is the ***frequency response*** of the ***relative displacement δ(t)*** of the ***seismic mass m*** with respect to the transducer case !

it can be seen on the curves that for every ***damping factor ζ***, the usable frequency range is for $\omega >> \omega_n$ where the gain is $\delta_0/U_0 \approx 1$ and the phase delay is $\varphi \approx -180°$ **…** in *phase opposition*

these conditions together imply that when the machine oscillation *u(t)* is at its maximum point U_0 the relative displacement *δ(t)* of the seismic mass *m* is at its minimum point $-\delta_0$ …

VIBRATION measurement techniques …

This means the *seismic mass m* actually «stopped moving» with respect to the absolute reference frame: in these conditions, the *seismic mass m* realizes the *fixed reference point* for vibration measurements !!

Operative considerations:

- To make the condition **$\omega >> \omega_n$** really usable, we need to design a transducer with a *very low* $\omega_n = \sqrt{\frac{k}{m}}$ which implies *$k \rightarrow$ very small* and *$m \rightarrow$ very big* !

- It is quite hard to simultaneously realize these two conditions (a very soft spring that bears a big mass !!)

- Moreover, a *big seismic mass* ***m*** firmly applied on the *machine mass* ***M*** causes a «change of the vibration modes» and, therefore, induces an ***insertion error*** !

Few transducers realize the conditions of above : the **vibrometer** and the **seismograph** …

In practice, it would be much better to make measurements with a transducer made with a «*small mass m*» and a «*high stiffness k*»

This implies a ***high natural frequency*** $\omega_n = \sqrt{\frac{k}{m}}$ and making measurement at frequencies **$\omega << \omega_n$**

ACCELERATION measurement techniques …

Because the gain δ_0/U_0 is quite low for $\omega << \omega_n$ we can not directly measure the *displacement u(t)* in these conditions and we have to *change the **measurand*** at the transducer input !

This implies to change transducer type …

In fact, being $\ddot{u}(t) = -\omega^2 \cdot U_0 sen\omega t$ for the amplitudes it results : $\ddot{U}_o = \omega^2 U_o$ $\quad U_o = \dfrac{\ddot{U}_o}{\omega^2}$

which substituted in the *frequency response* of the vibrometer results :

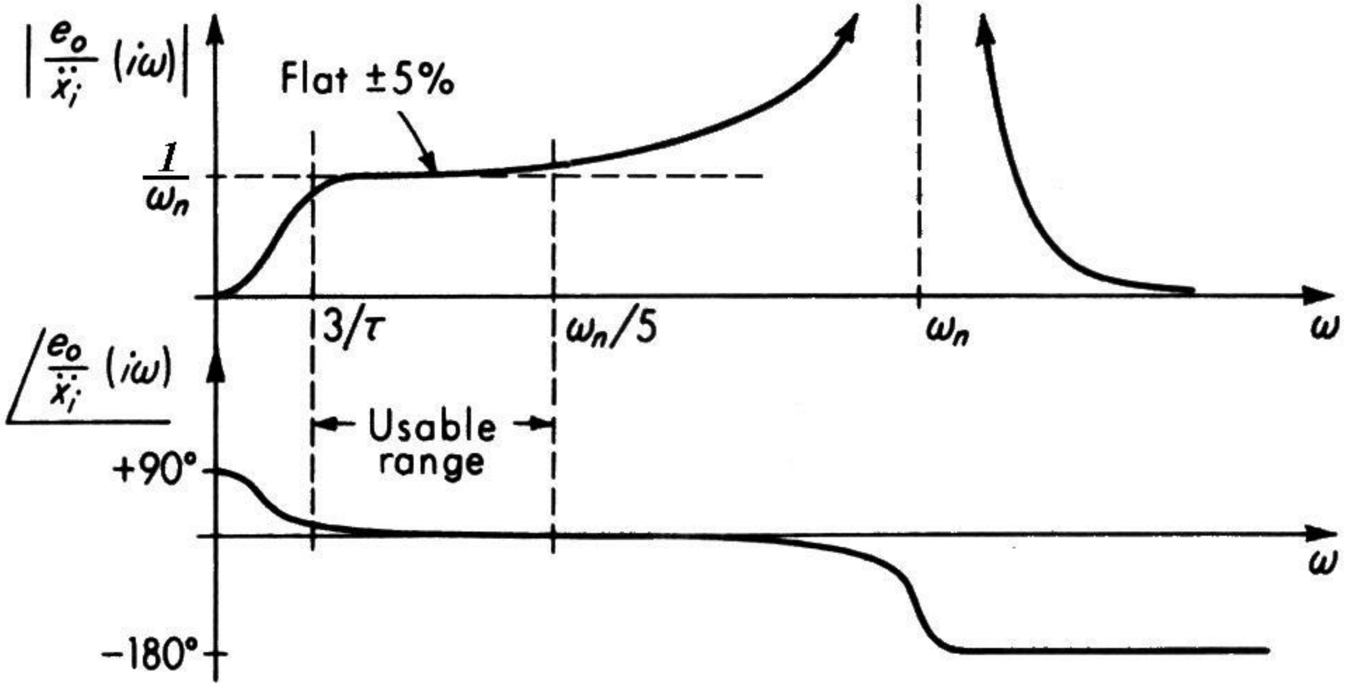
$$\delta_o = U_o \frac{\omega^2}{\omega_n^2} \cdot \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}} = \frac{\ddot{U}_o}{\omega^2} \cdot \frac{\omega^2}{\omega_n^2} \cdot \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

We need to employ a sensor made with the same constitutive elements of the vibrometer (*m; c; k*), but measuring the *drag acceleration* $\ddot{u}(t)$

Such a transducer is the ***accelerometer*** which has a ***frequency response*** :

$$G = \frac{\delta_o}{\ddot{U}_o} = \frac{1}{\omega_n^2} \cdot \frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

ACCELERATION measurement techniques …



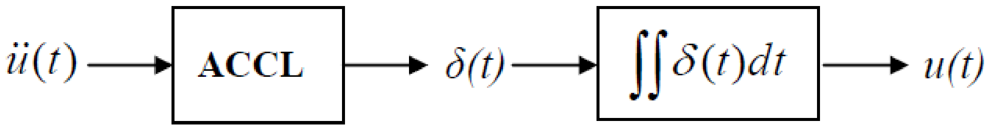
Piezoelectric accelerometer frequency response

The usable frequency range is now at the left of the *resonance frequency* ω_n !

To extend the frequency band we have to increase ω_n however, this implies a decrease

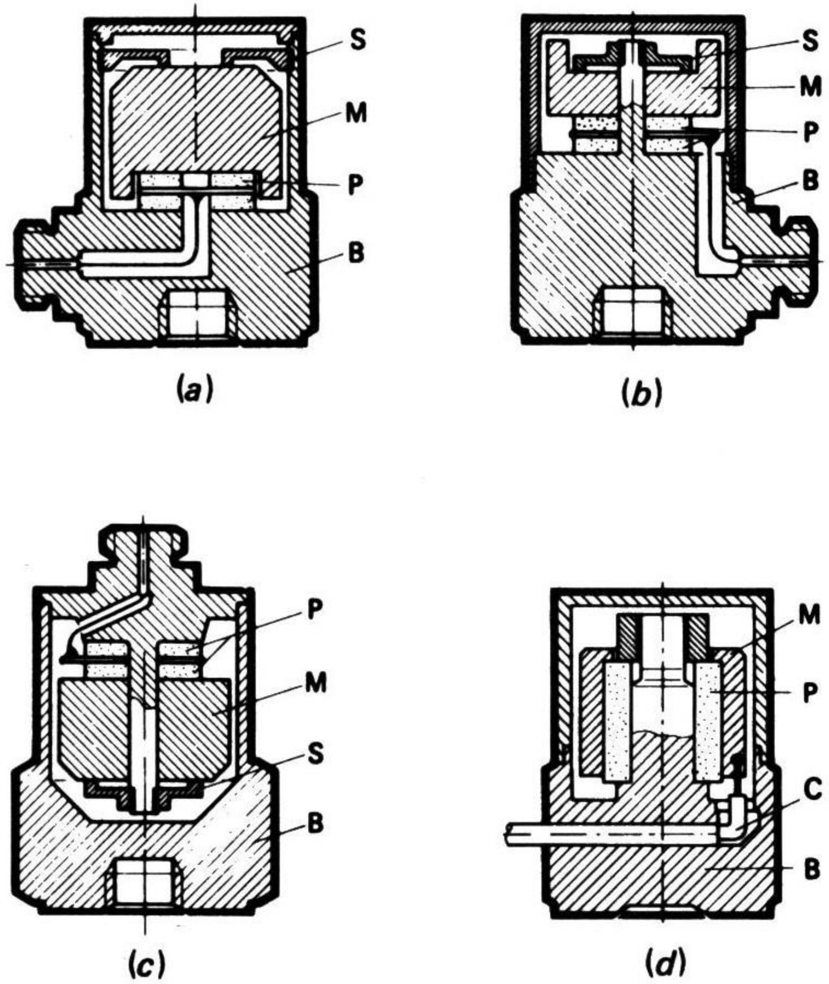
of the *gain* $\dfrac{\delta_o}{\ddot{U}_o}$ or the sensor *sensitivity* !

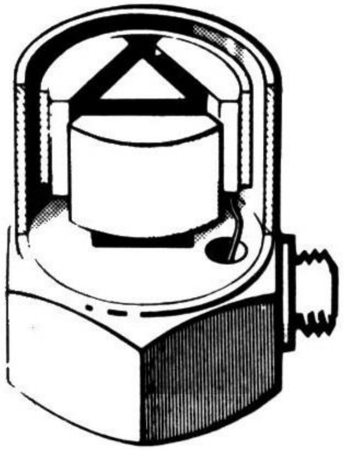
If we wish to get the displacements *u(t)* with such a sensor, we have to proceed with a double integration of the accelerometer signal …



<u>Piezoelectric accelerometers</u> are still the best acceleration transducers available on the market, followed by *capacitive* and *semiconductor accelerometers* …

ACCELERATION measurement techniques …





Piezoelectric accelerometer designs.

(*a*) Peripheral-mounted compression design.

(*b*) Center-mounted compression design.

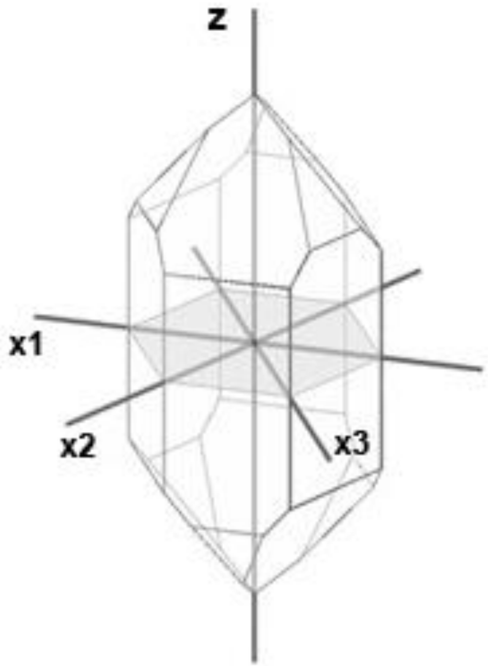
(*c*) Inverted center-mounted compression design.

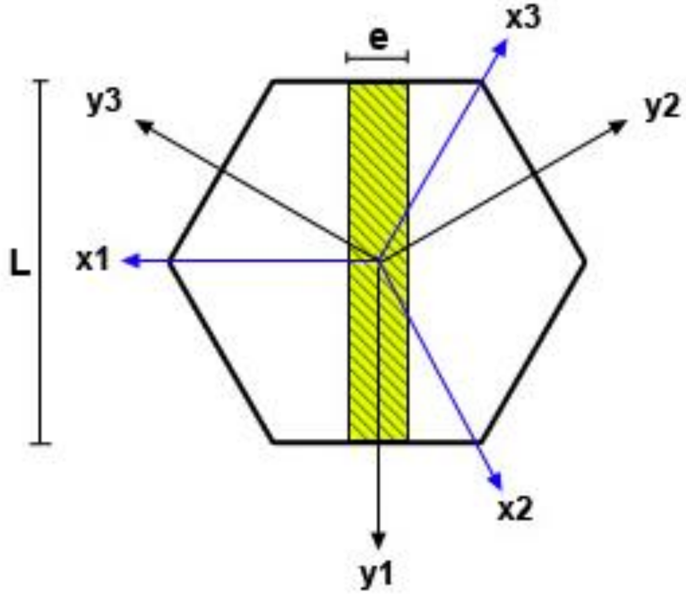
(*d*) Shear design.

S: Spring M: Mass P: Piezoelectric element B: Base C: Cable

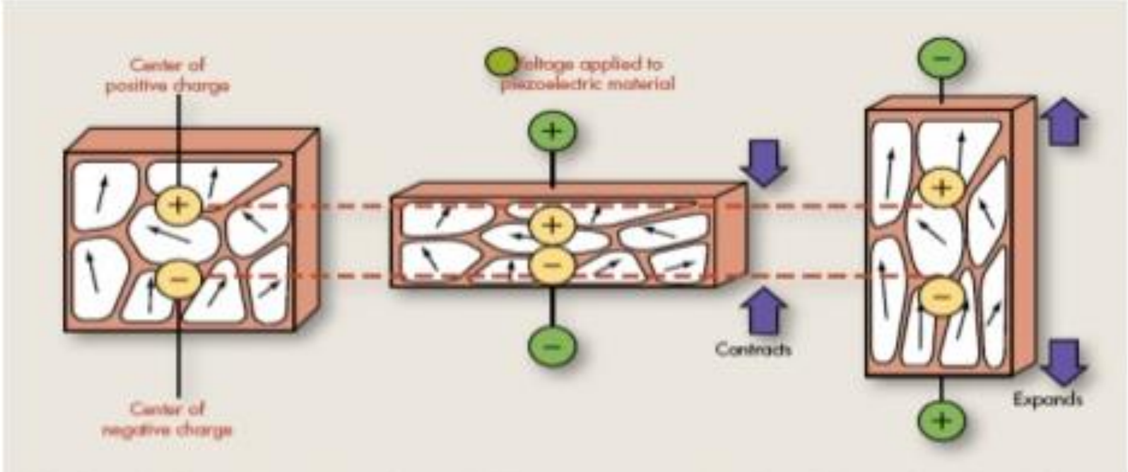
(*Courtesy Bruel and Kjaer Instruments, Marlboro, Mass.*)

ACCELERATION measurement techniques …

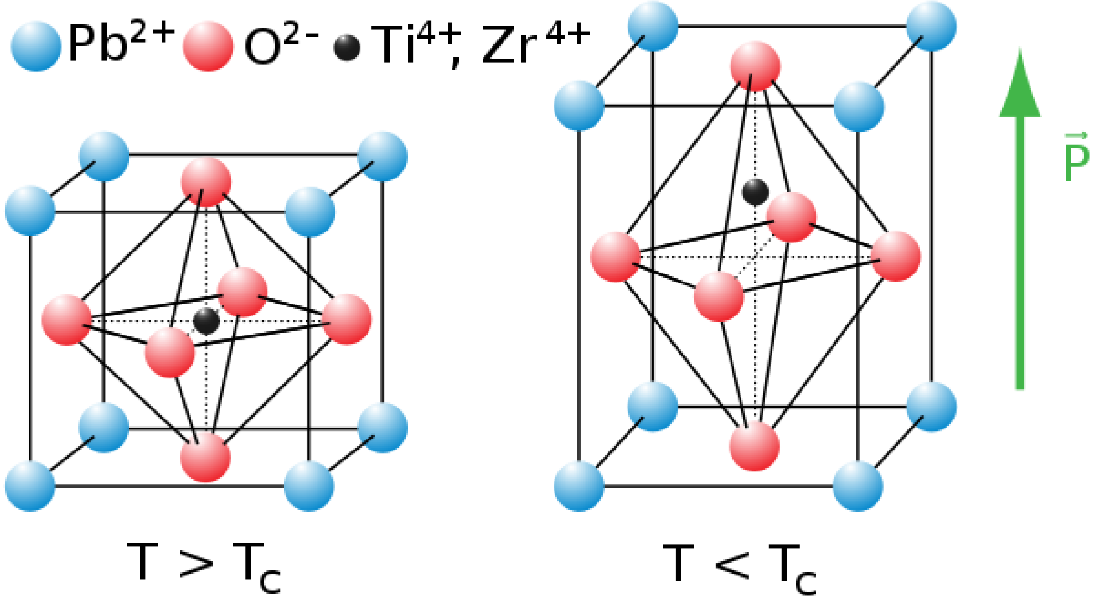




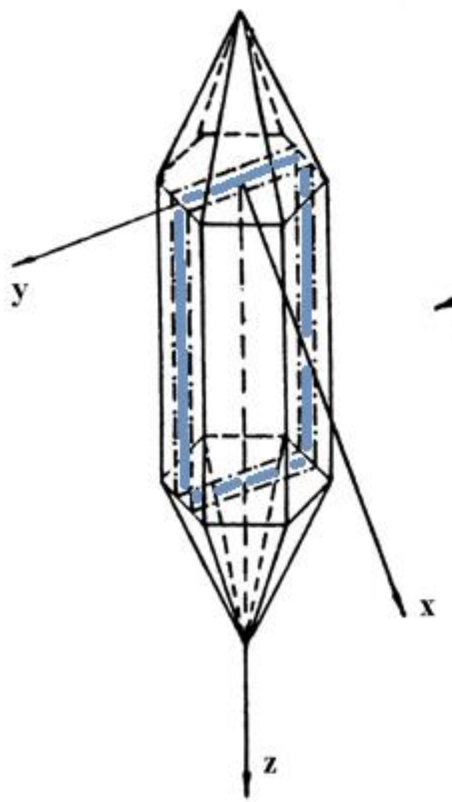
Although the most common ***piezoelectric crystal*** is **quartz**, to realize ***piezoelectric transducers,*** titanates of barium ($BaTiO_3$) are generally used, or the so-called ***PZT*** (from Lead - Zirconium - Titanium, solid solutions of $PbZrO_3$ and $PbTiO_3$)

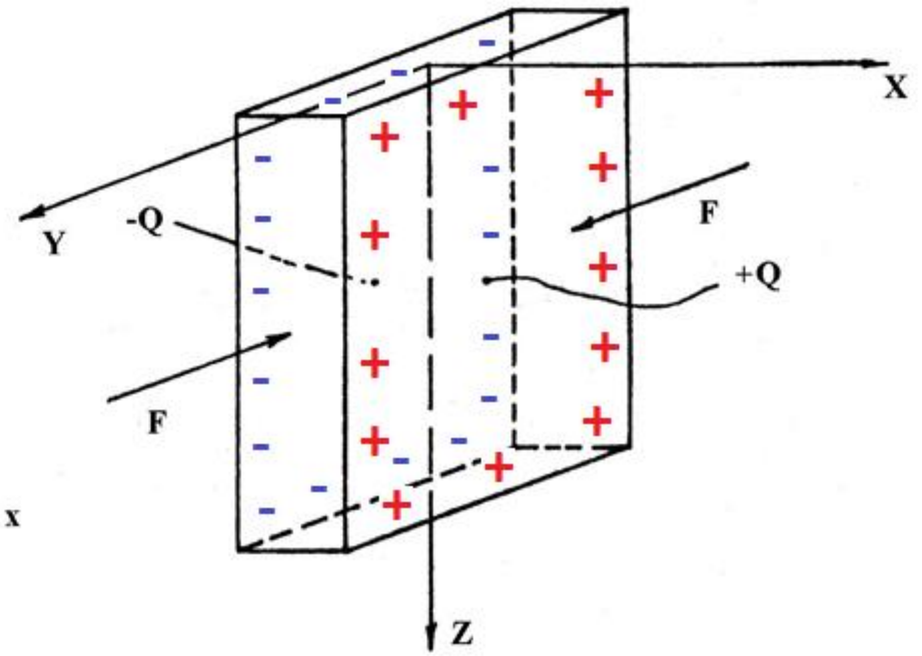


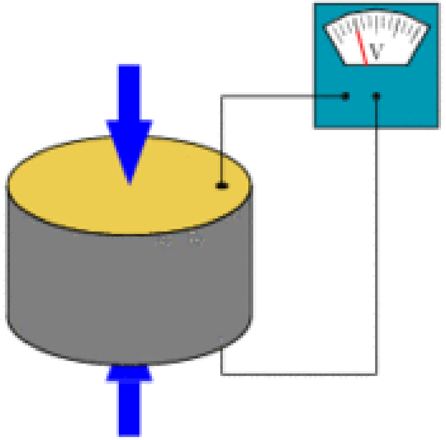
1. The piezoelectric effect causes crystal materials like quartz to generate an electric charge when the crystal material is compressed, twisted, or pulled. The reverse also is true, as the crystal material compresses or expands when an electric voltage is applied.



ACCELERATION measurement techniques …







Piezoelectric quartz crystallizes in the hexagonal system. A slice is cut and extracted from the crystal such as in the figure

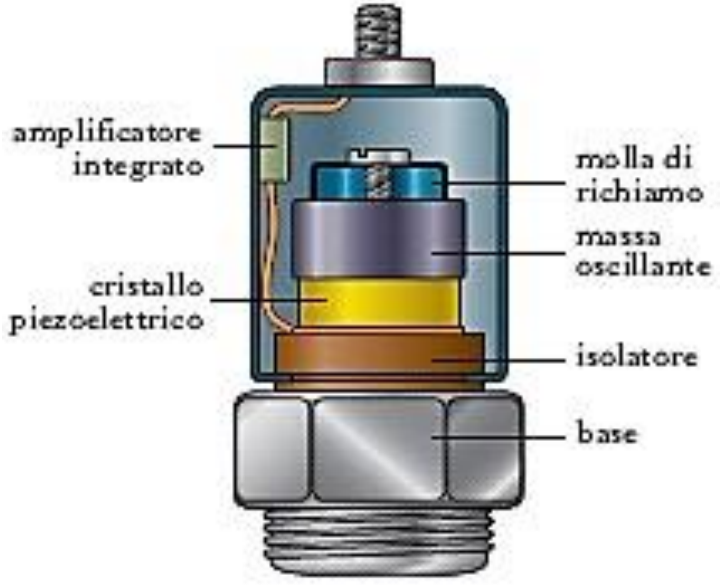
When applying a ***force F*** parallel to the y axis, a ***charge ±Q*** appears on the x faces

The charge stays as long as force is applied on the y faces: $Q = k_{ij} \cdot F$
k_{ij} are the *piezoelectric constants*

Because of the form they are cut, *piezoelectric crystals* can be considered as ***capacitors*** with a capacity : $C = \varepsilon_0 \varepsilon_r S / d$
The charge then is : $Q = C \cdot V$ and the ***graduation curve*** is :

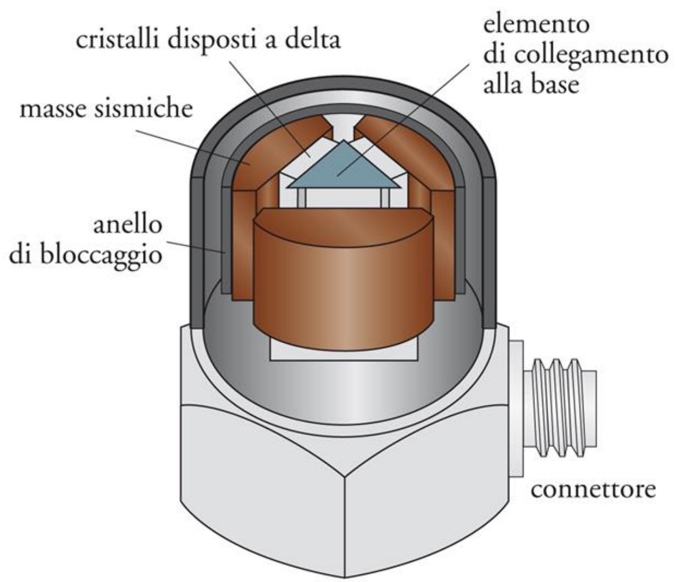
$$V = \frac{Q}{C} = \frac{k_{ij} \cdot F}{\varepsilon_0 \varepsilon_r S / d} = \frac{k_{ij} \cdot m\ddot{y}}{\varepsilon_0 \varepsilon_r S / d} = \frac{d k_{ij} m}{\varepsilon_0 \varepsilon_r S} \cdot \ddot{y}$$

ACCELERATION *measurement techniques …*

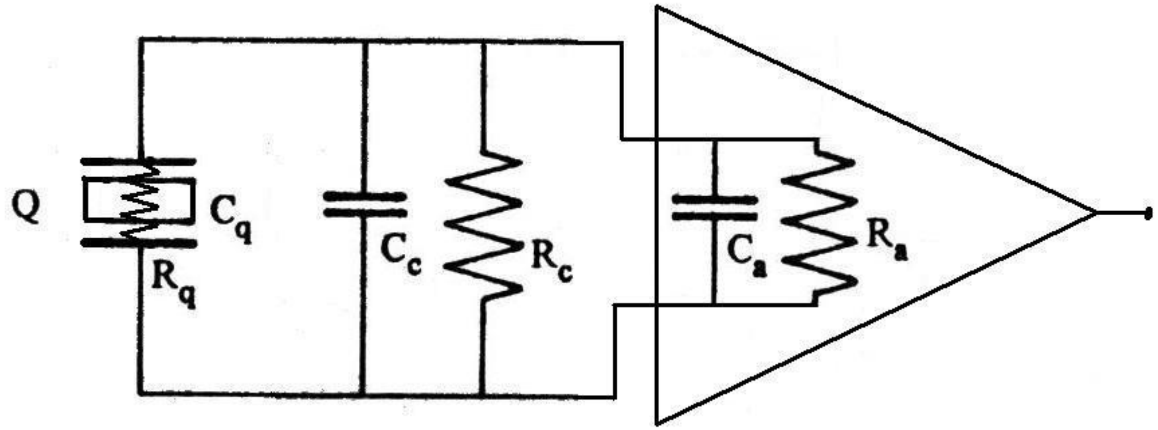


m is the *seismic mass* that during vibration presses on the piezoelectric crystal with the *inertial forces* $F = m\ddot{y}$

Note that *m* is mechanically *pre-charged* to the crystal so to output a signal also for negative inertial forces …
c and *k* are the crystal internal *damping* and *stiffness* …



Piezoelectric transducers, unfortunately, have a very *high output impedance* (10^9 Ω) which makes them very *poor signal generators*.
To actually read the signal proportional to the acceleration $\ddot{y}$ we have to employ special amplifiers, with very high input impedance (10^{12} Ω): the ***charge amplifiers !***



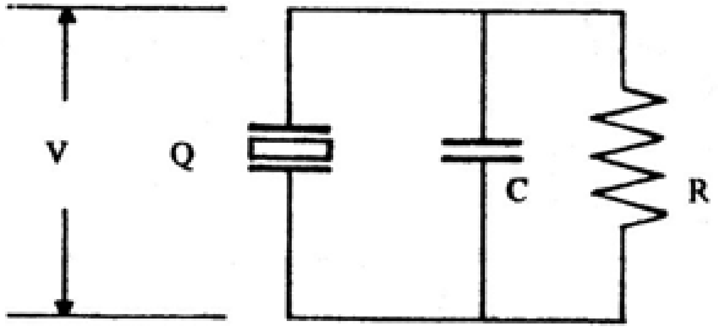
ACCELERATION measurement techniques …

Many elements play a role in this delicate *transducer–amplifier coupling* :

C_q = piezoelectric quartz capacity R_q = piezoelectric quartz output resistance
C_c = leads coupling capacity R_c = leads resistance
C_a = amplifier input capacity R_a = amplifier input resistance

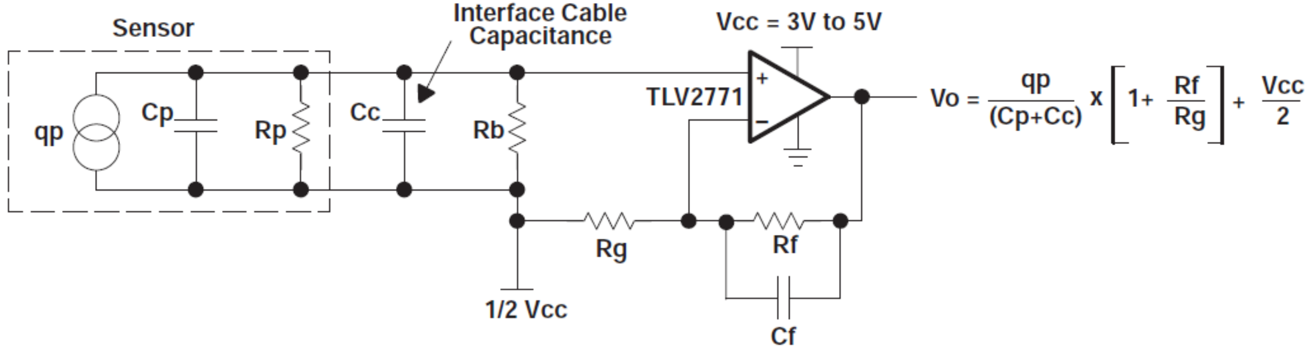
We have all passive elements in parallel :

$$C = C_q + C_c + C_a \qquad \frac{1}{R} = \frac{1}{R_q} + \frac{1}{R_c} + \frac{1}{R_a} \qquad R = \frac{R_q R_c R_a}{R_c R_a + R_q R_a + R_q R_c}$$

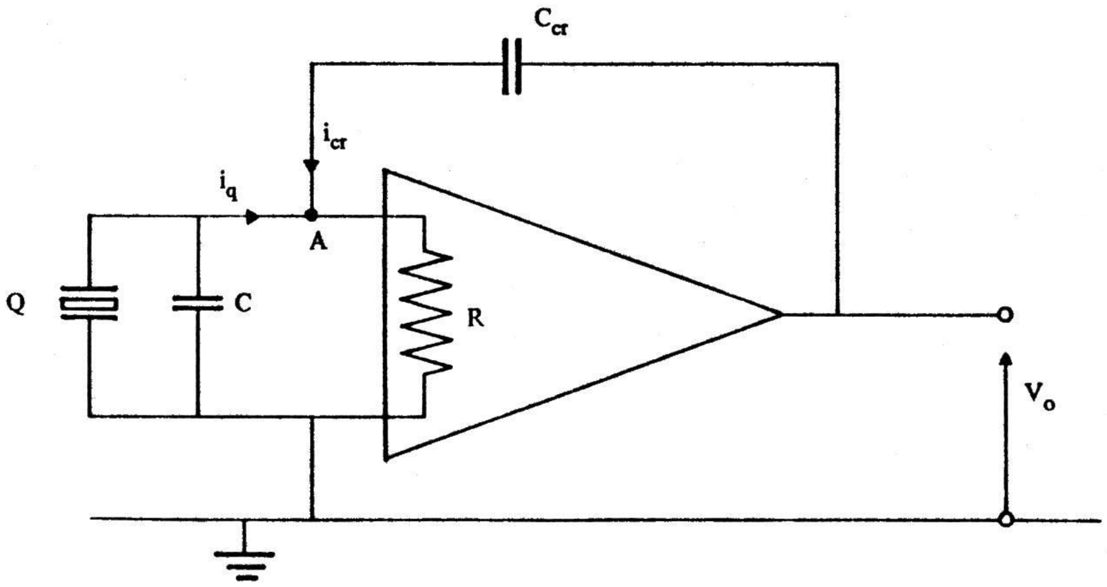


It discharges exponentially, like a 1° order electrical system : $V(t) = V_o e^{-\frac{t}{RC}}$

Therefore, *charge amplifiers* are made with *FET input stage inverting AO* and a *feedback capacitor* :

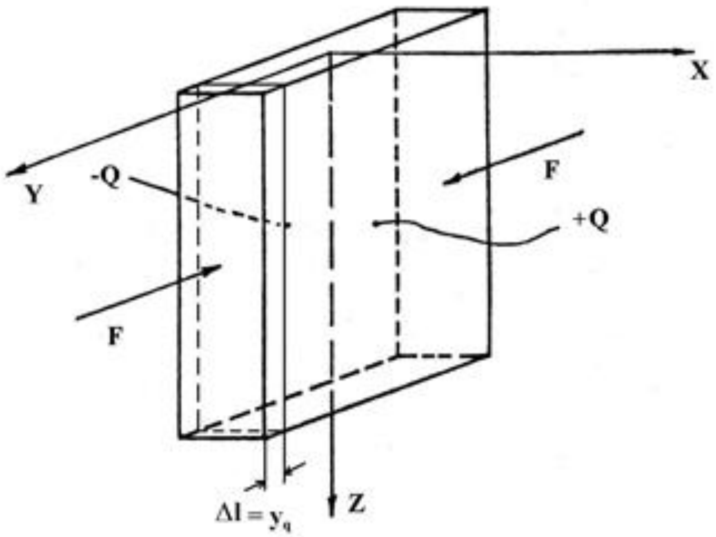


ACCELERATION *measurement techniques …*



Note that the graduation curve $V = \frac{dk_{ij}m}{\varepsilon_0 \varepsilon_r S} \cdot \ddot{y}$ was derived

without taking into account that quartz is still an *elastic element* (with a *very high* *Young's modulus E*) and that the *charge Q* becomes available on the faces of the crystal due to an internal electrical asymmetry, caused by the very small *deformation ε* of the crystal.
The *deformation ε* in turn is caused by the *inertia forces* of the *seismic mass m* which acts during vibration



$$Q = k_{ij} \cdot F = k_{ij} \cdot S\sigma = k_{ij} \cdot S \cdot E\varepsilon = k_{ij} \cdot SE \cdot \frac{\Delta l}{l}$$

$$Q = \frac{k_{ij}SE}{l} \cdot y_q$$

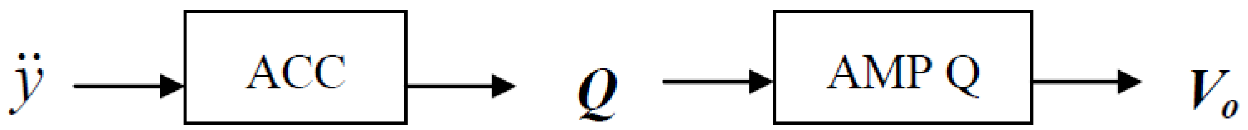
On the node A in the figure above, we have: $i_q = -i_f$

$$i_q = \frac{dQ}{dt} = K \cdot \dot{y}_q$$ being $$\frac{k_{ij}SE}{l} = \cos t = K$$

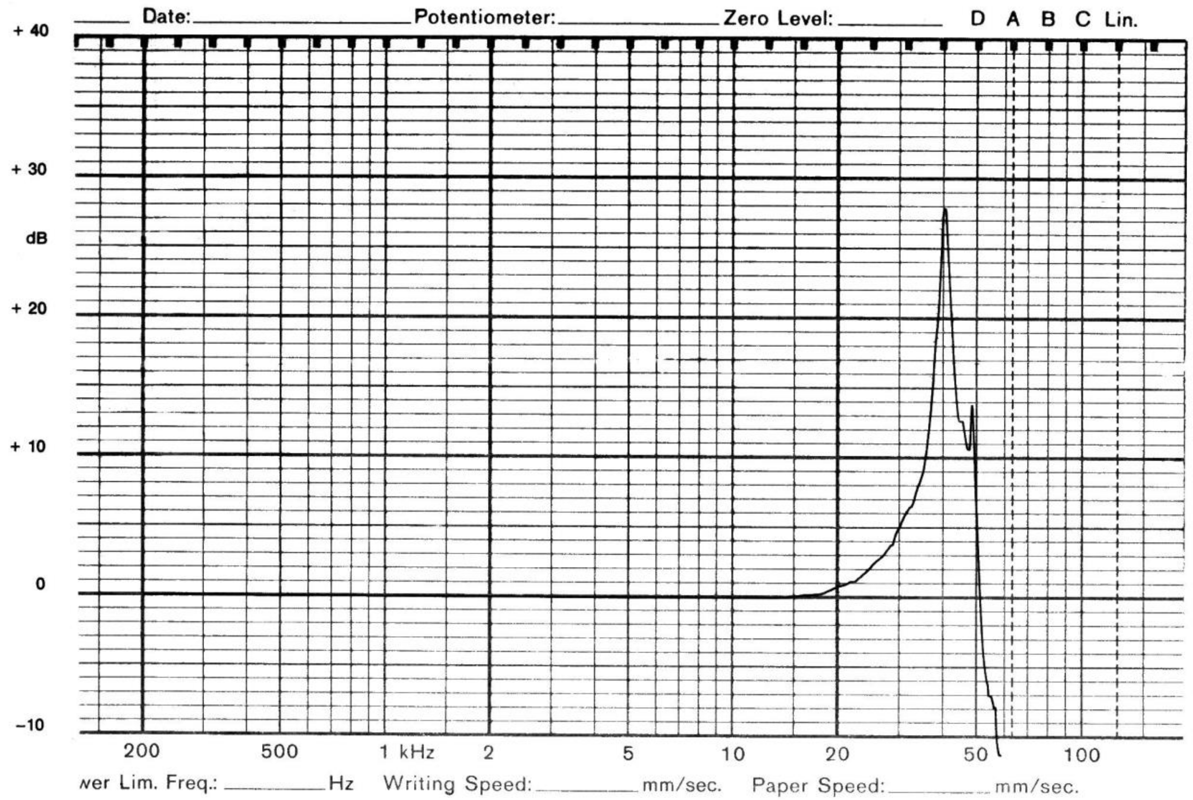
$$i_f = C_f \frac{dV_o}{dt} = C_f \cdot \dot{V}_o$$ therefore: $$K \cdot \dot{y}_q = -C_f \cdot \dot{V}_o$$

***ACCELERATION* measurement techniques …**

The graduation curve of the integrated system of the piezoelectric accelerometer and the charge amplifier finally is:



$$V_o = -\frac{K}{C_f} \cdot y_q = -\frac{k_{ij} SE}{l C_f} \cdot y_q$$



At any rate, piezoelectric accelerometers have a determined *time constant* and therefore are *NOT suited to measure a constant acceleration* (ω = 0) !

Piezoelectric accelerometers are factory calibrated one by one !

Manufacturer provide the user with the *transducer sensitivity in [pC/ms^{-2}]* and the *frequency response chart*